

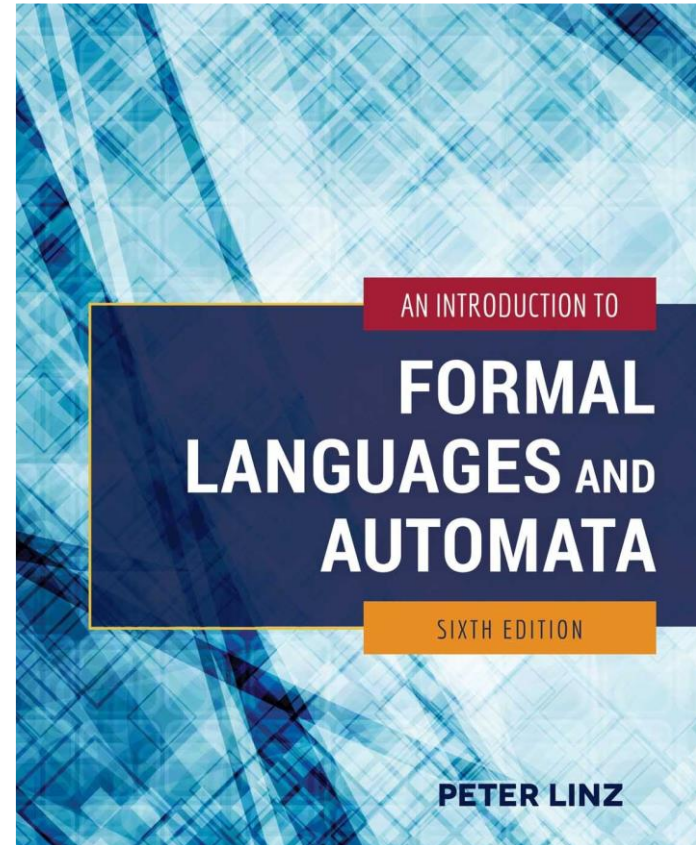
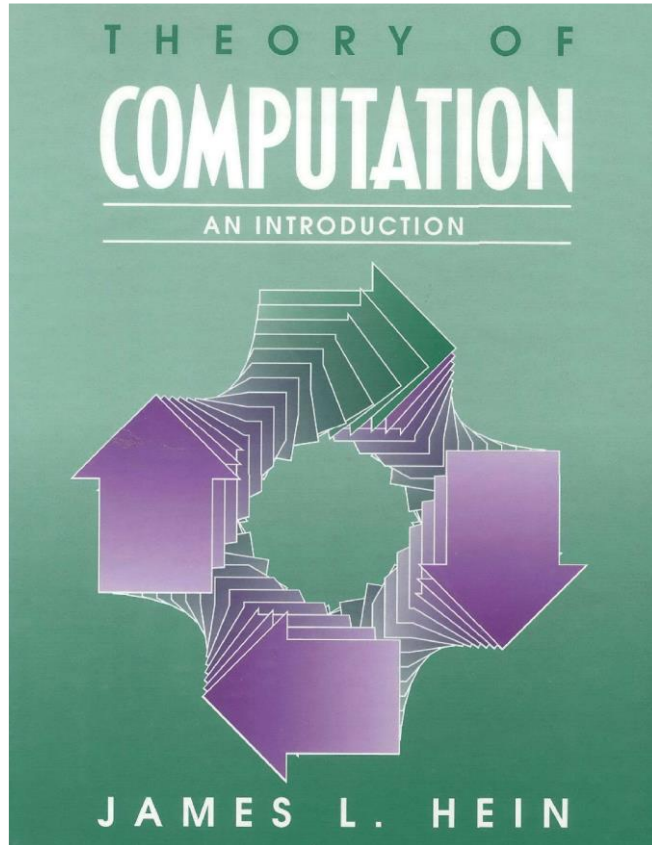
# Automata and Formal Languages

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Lecture 09

# Books

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# PowerPoint

<http://www.bu.edu.eg/staff/ahmedaboalatah14-courses/14767>

The screenshot shows a web interface for Benha University. At the top, there is a blue header with the university logo, the name 'Benha University', and a welcome message for 'Ahmed Hassan Ahmed Abu El Atta' with a 'Log out' link. Below the header, a navigation menu on the left lists various university services. The main content area displays course details for 'Automata and Formal Languages' taught by 'Ass. Lect. Ahmed Hassan Ahmed Abu El Atta'. The details are presented in a table with blue headers and white content. A 'Course password' section is also visible. On the right side, there are social media icons and a vertical toolbar with various icons.

Benha University

Staff Search: **Welcome: Ahmed Hassan Ahmed Abu El Atta (Log out)**

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Ass. Lect. Ahmed Hassan Ahmed Abu El Atta :: Course Details:  
Automata And Formal Languages [add course](#) | [edit course](#)

Course name	Automata and Formal Languages
Level	Undergraduate
Last year taught	2018
Course description	Not Uploaded
Course password	
Course files	<a href="#">add files</a>
Course URLs	<a href="#">add URLs</a>
Course assignments	<a href="#">add assignments</a>
Course Exams & Model Answers	<a href="#">add exams</a>

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Home

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About

Courses

Publications

Inlinks(Competition)

Theses

Reports

Published books

Workshops / Conferences

Supervised PhD

Supervised MSc

Supervised Projects

Education

Language skills

Academic Positions

Administrative Positions

Google

RG

in

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+

+

+

+

+

+

+

+

+

(edit)

# Pumping lemma

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PUMPING LEMMA

# Agenda

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- Pumping lemma1
- Example 1
- Pumping lemma2
- Example 2
- Example 3

# Non-regular languages

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(PUMPING LEMMA)

Non-regular languages

$$L = \{a^n b^n : n \geq 0\}$$

$$L = \{vv^R : v \in \{a, b\}^*\}$$

Regular languages

$$a^*b$$

$$b^*c + a$$

$$b + c(a + b)^*$$

*etc...*

How can we prove that a language  $L$  is not regular?

Prove that there is no DFA or NFA or RE that accepts  $L$

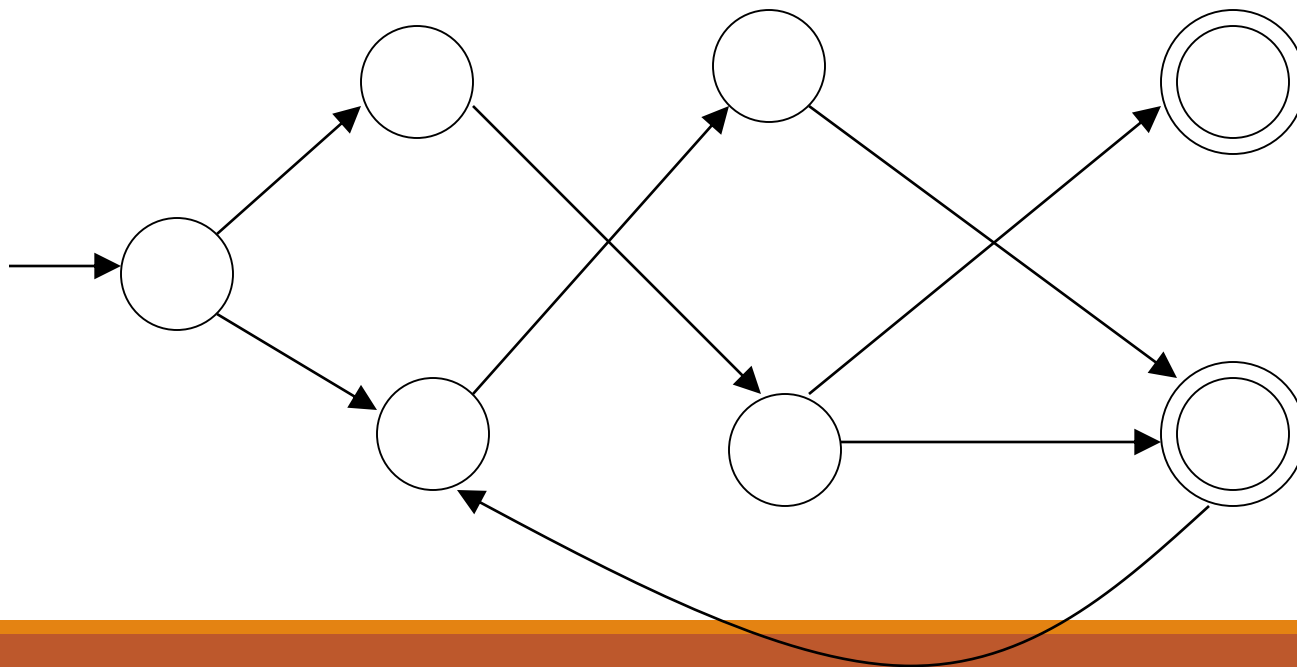
**Difficulty:** this is not easy to prove  
(since there is an infinite number of them)

**Solution:** use the Pumping Lemma !!!



Take an **infinite** regular language  $L$   
(contains an infinite number of strings)

There exists a DFA that accepts  $L$

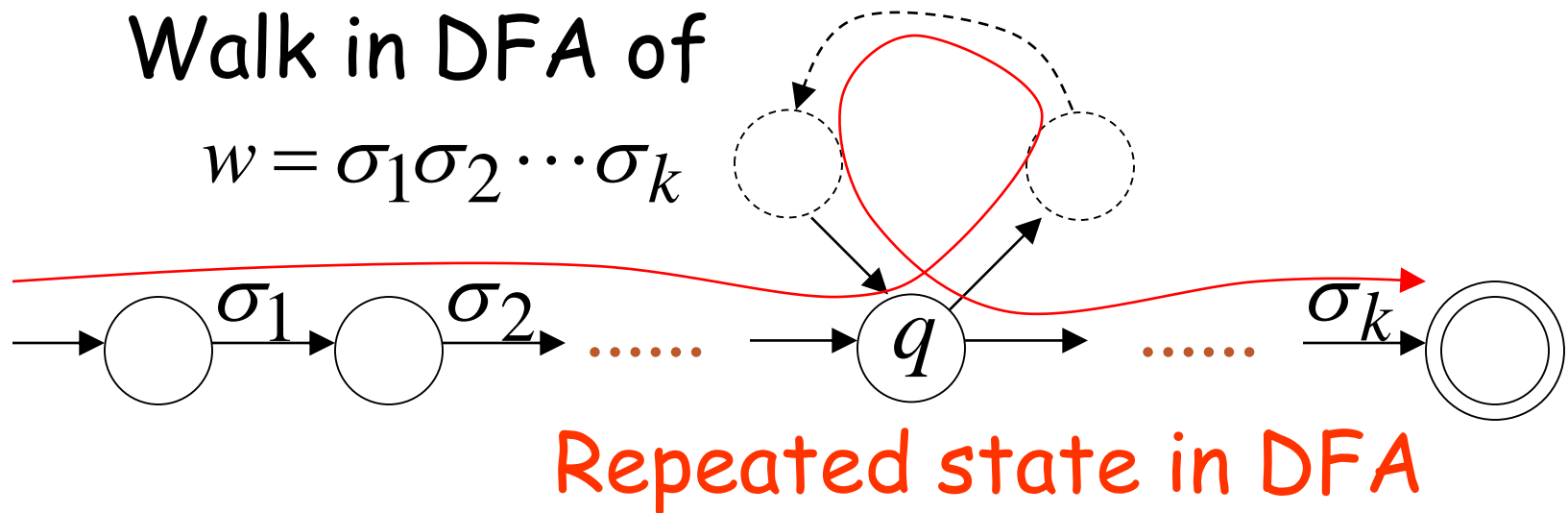


$m$   
states

Take string  $w \in L$  with  $|w| \geq m$

(number of  
states of DFA)

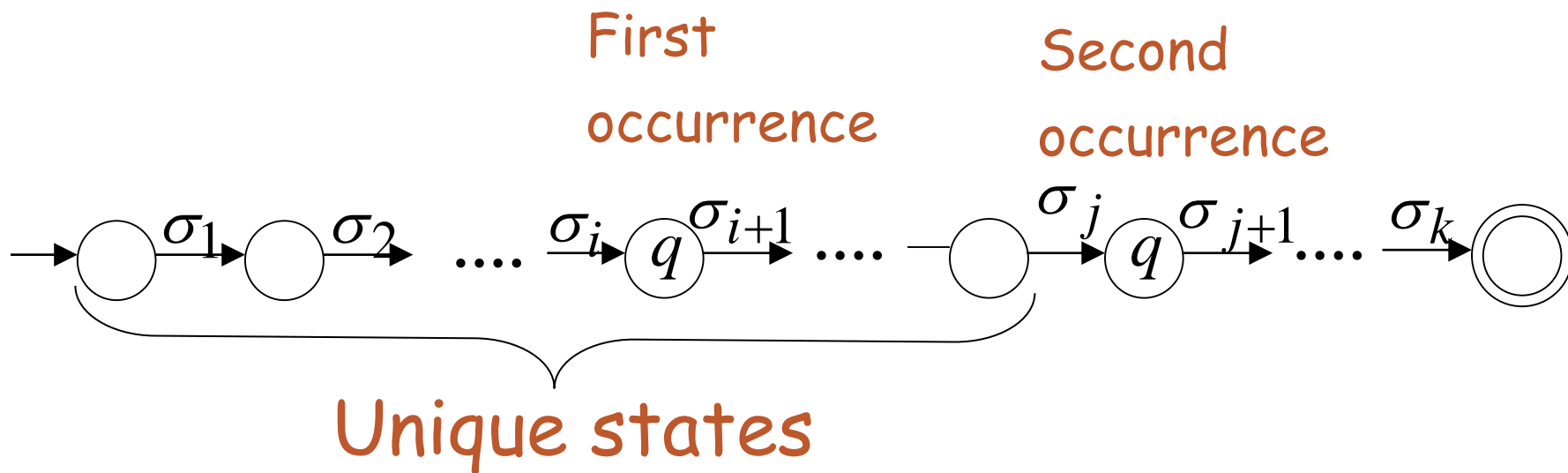
then, at least one state is repeated  
in the walk of  $w$



There could be many states repeated

Take  $q$  to be the first state repeated

One dimensional projection of walk  $w$  :

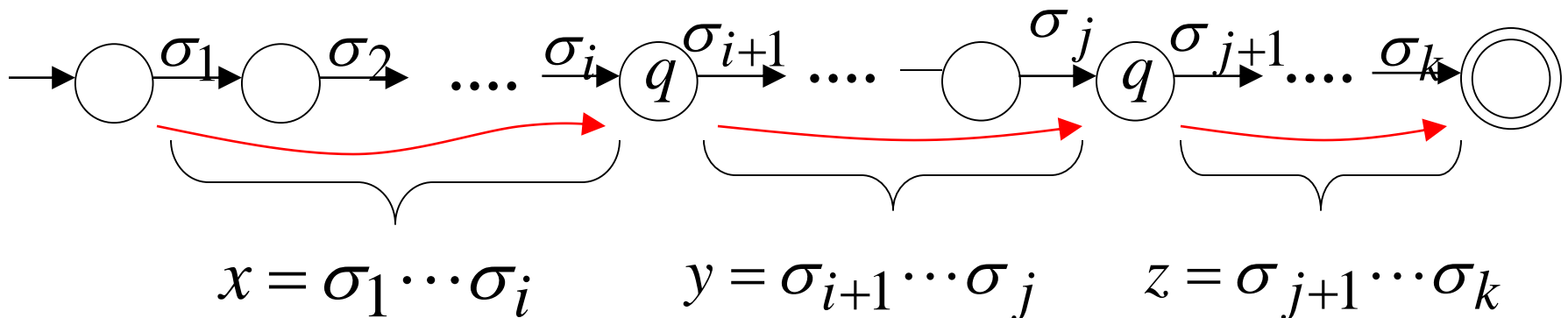


We can write  $w = xyz$

One dimensional projection of walk  $w$  :

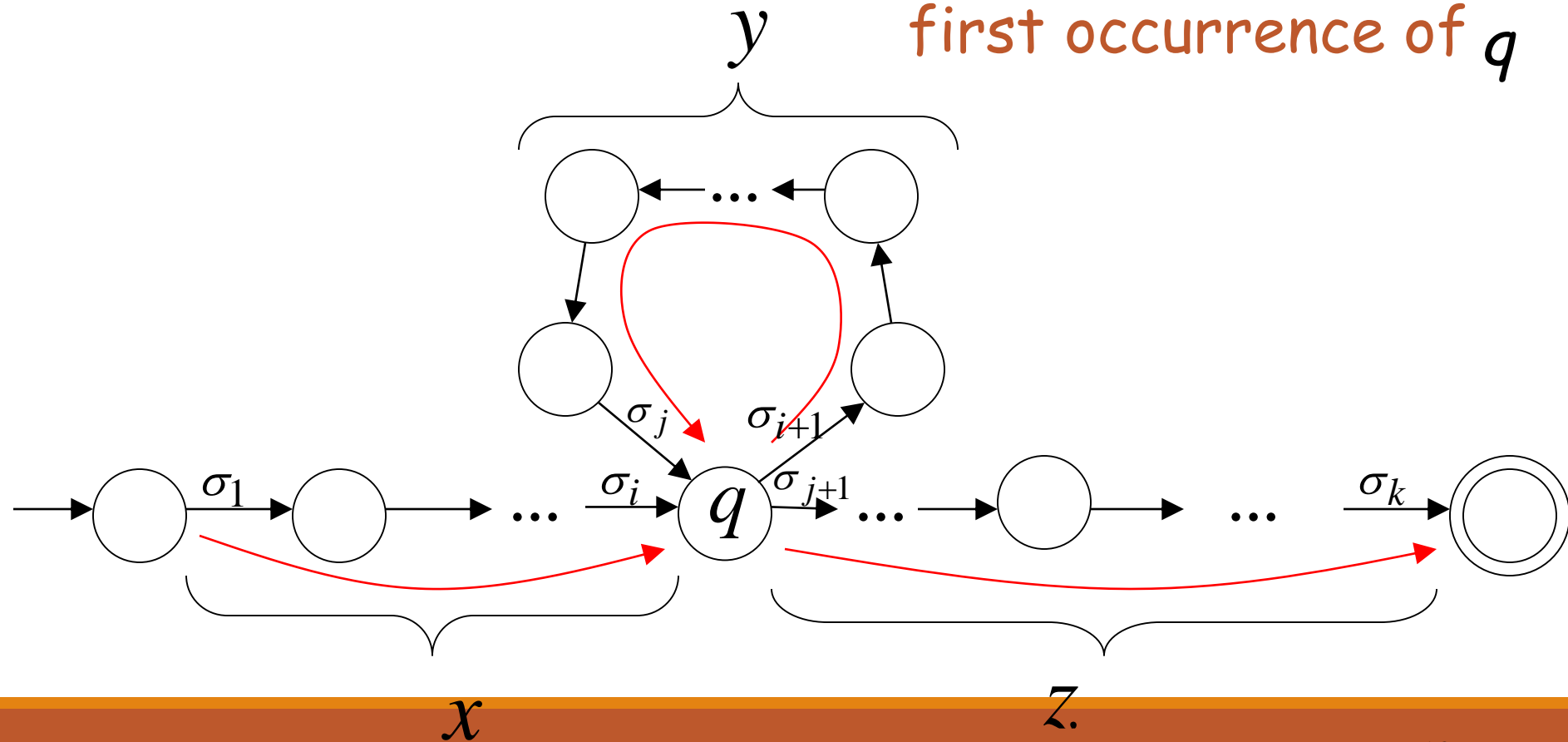
First  
occurrence

Second  
occurrence



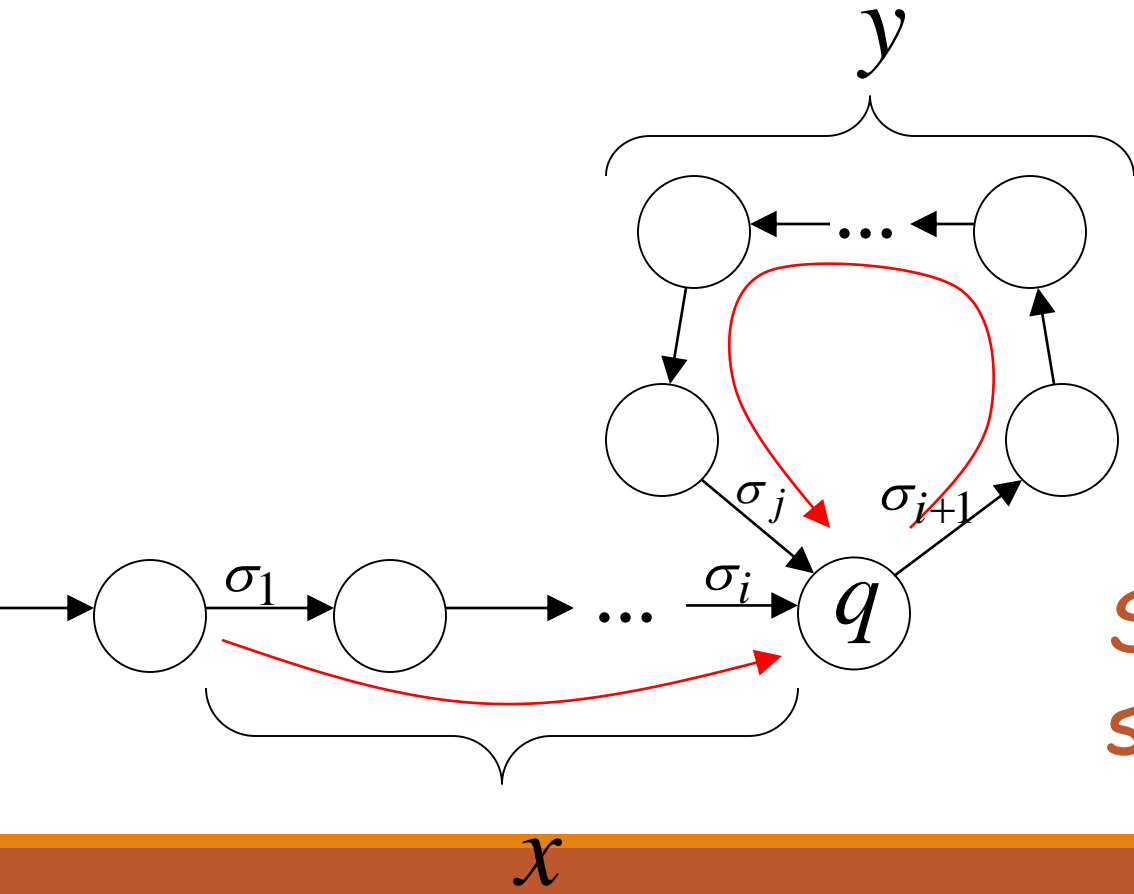
In DFA:  $w = x y z$

contains only  
first occurrence of  $q$



Observation:

length  $|x y| \leq m$  number of states of DFA

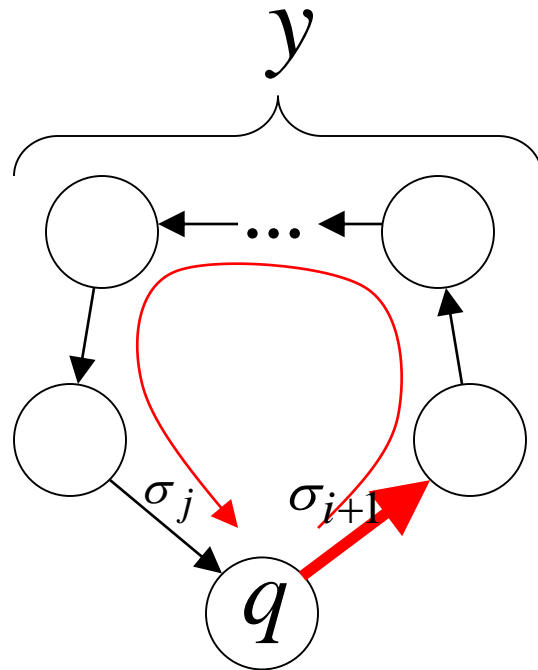


Unique States

Since, in  $xy$  no state is repeated (except  $a$ )

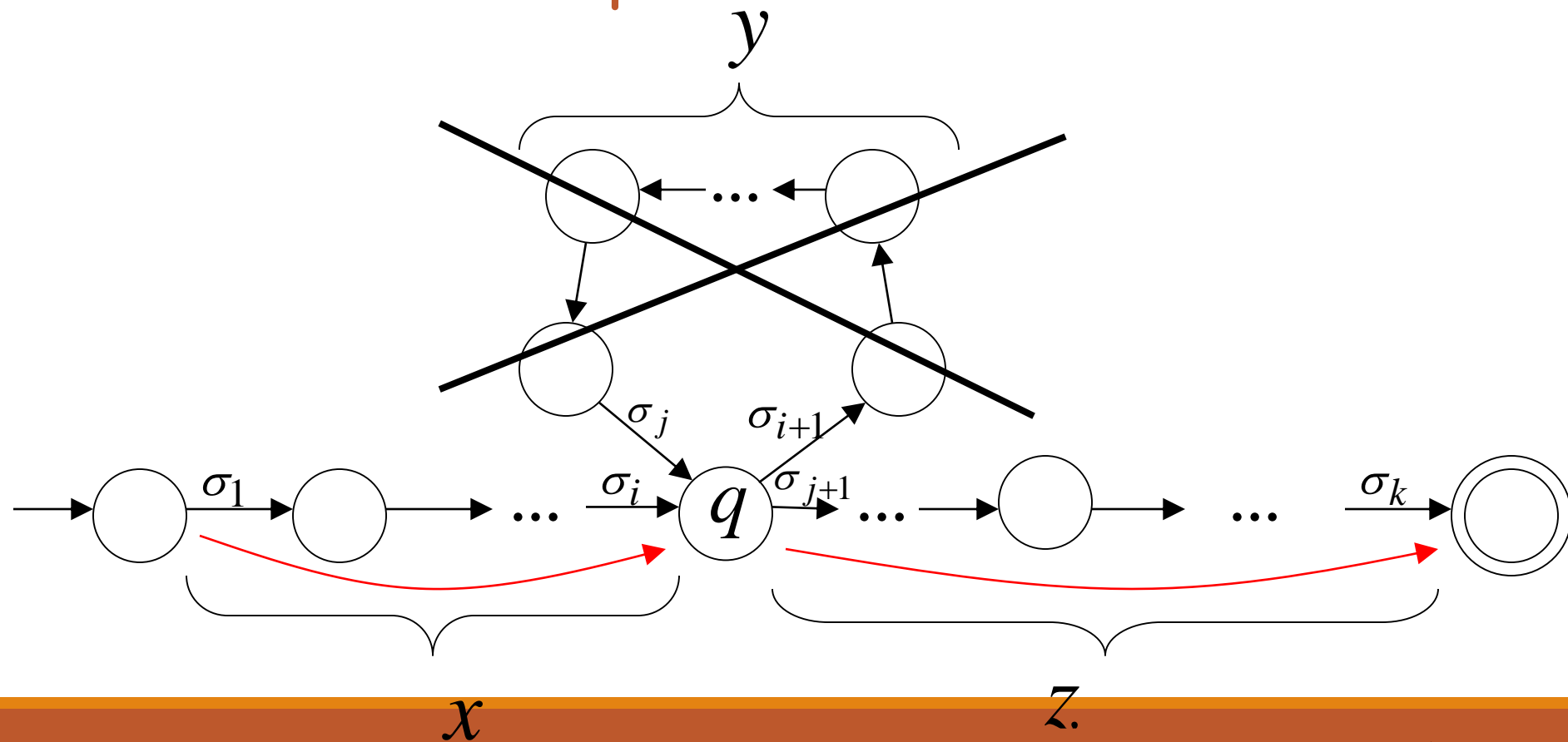
Observation:  $\text{length } |y| \geq 1$

Since there is at least one transition in loop



Additional string: The string  $xz$  is accepted

Do not follow loop

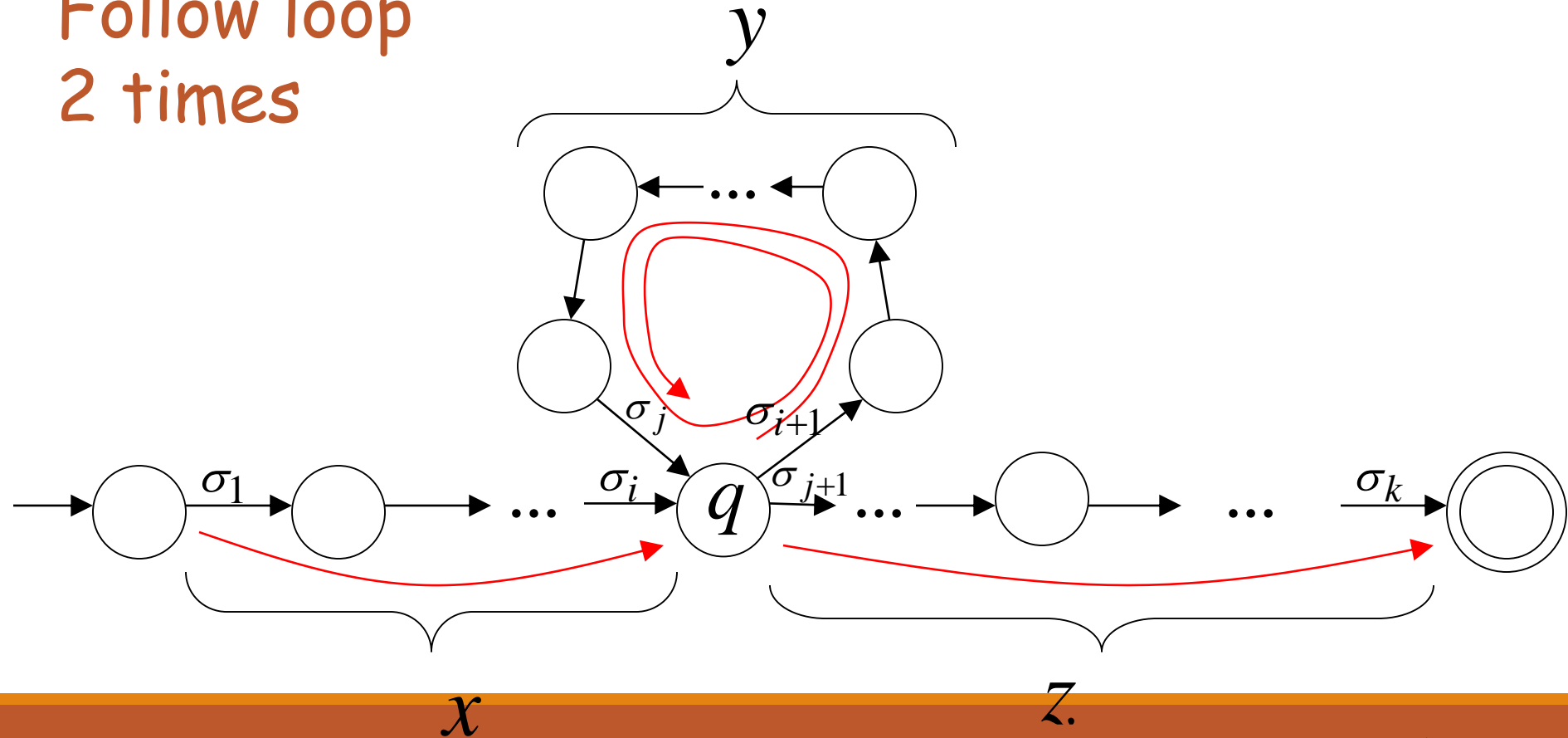




Additional string:

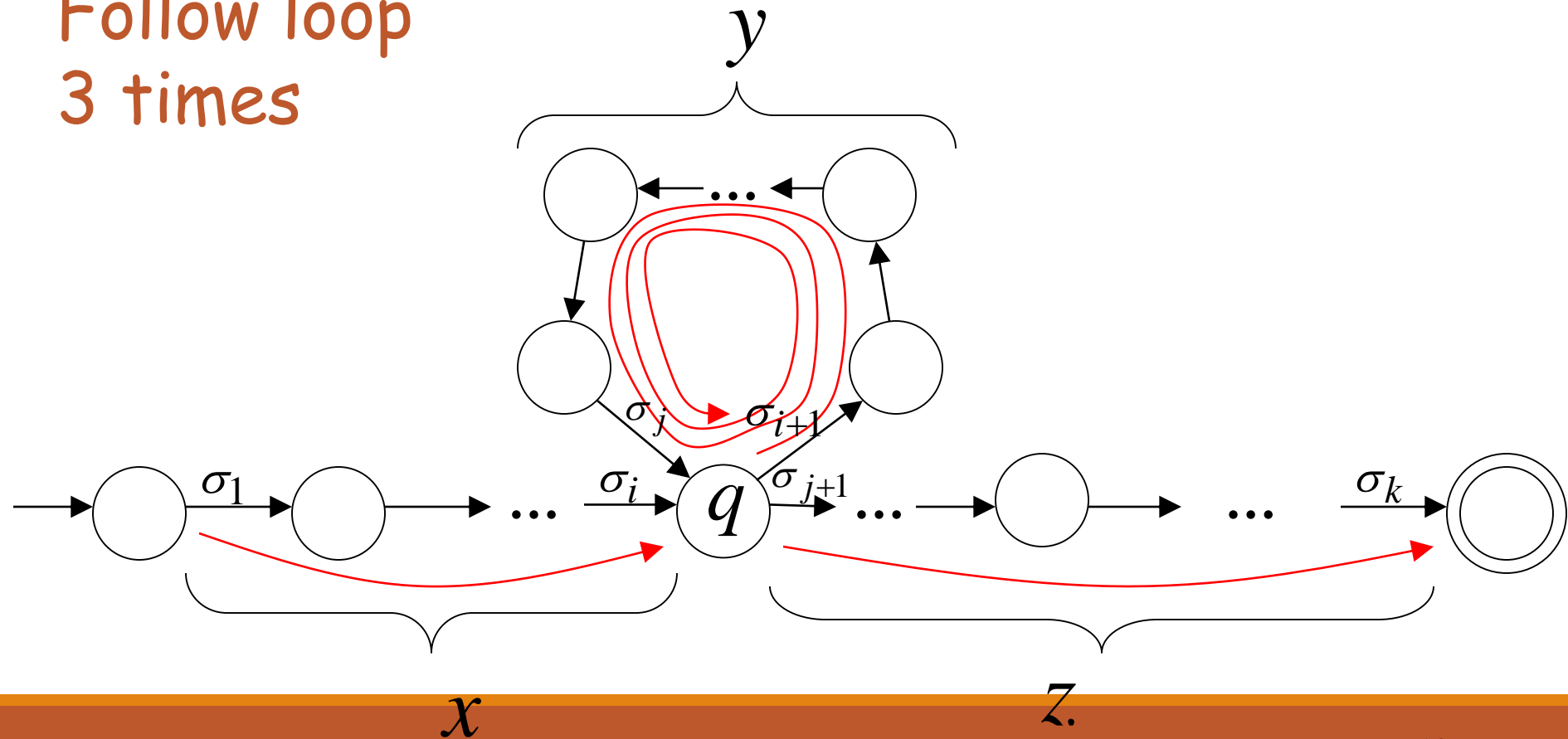
The string  $x y y z$   
is accepted

Follow loop  
2 times



Additional string: The string  $x y y y z$  is accepted

Follow loop 3 times

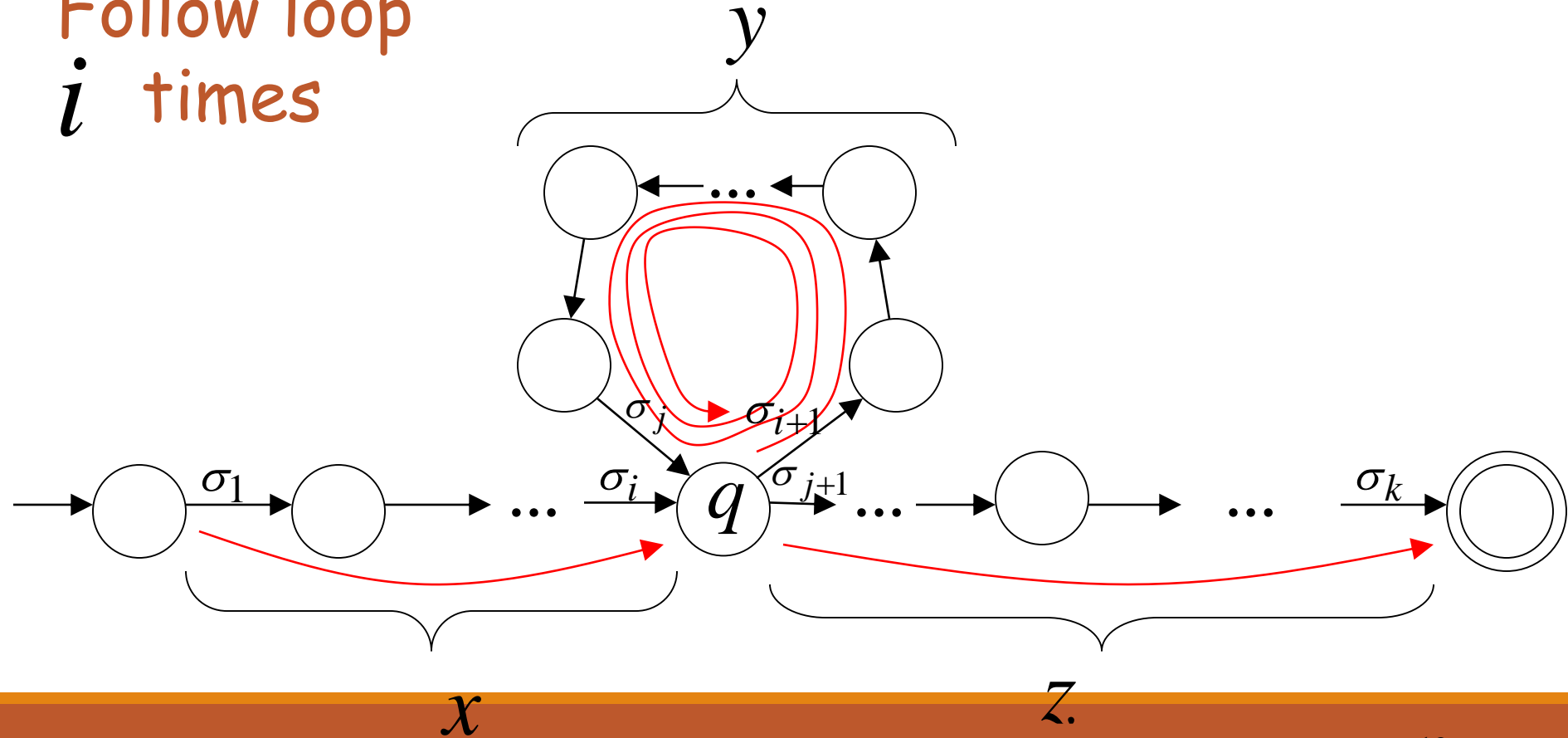


In General:

The string  
is accepted

$$x y^i z$$
$$i = 0, 1, 2, \dots$$

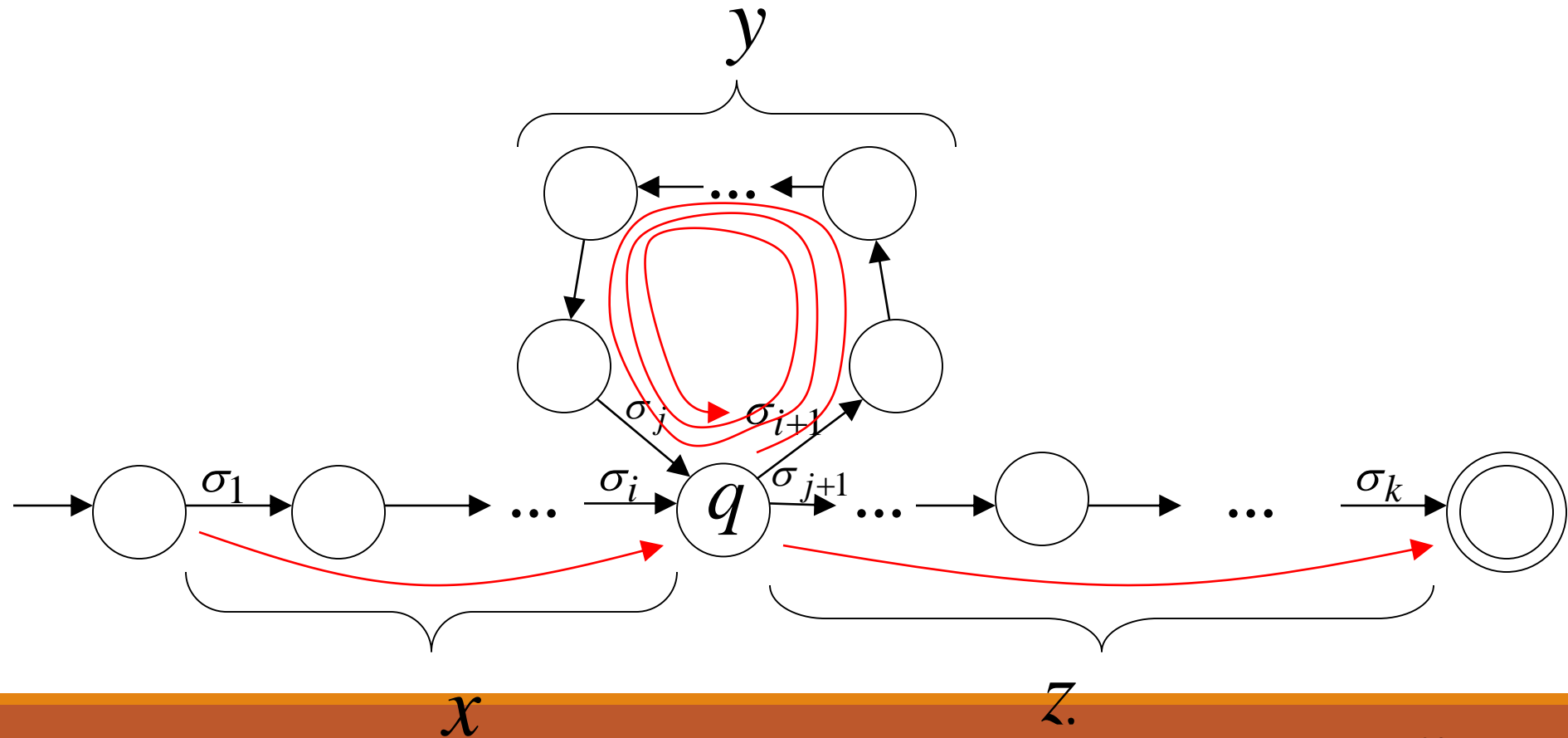
Follow loop  
 $i$  times



Therefore:

$$x y^i z \in L \quad i = 0, 1, 2, \dots$$

Language accepted by the DFA



# Pumping lemma(weak)

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Let  $L$  be an infinite regular language over  $A$ .  
then there exist strings

$x, y, z \in A^*$ , where  $y \neq \varepsilon$ ,

such that

$xy^iz \in L$  for all  $i \geq 0$ . such that

# Example 1

---

$$L = \{a^n b^n : n \geq 0\}$$

Let  $L$  be is regular and suppose  $L$  can be accepted by DFA  $(M)$ . Then there exist  $x, y, z \in A^*$ , let  $i=1$

By pumping lemma  $w = xyz = a^m b^m$

1.  $y$  consists of  $a$ 's.  $|xy|=k, y=a^r. xy=a^k$

$xy^2z = a^{m-r} \cdot a^{2r} \cdot a^{m-k} b^m = a^{m+r} b^m$  is not in  $L$

# Example 1

---

2.  $y$  consists of  $b$ 's.

$$xy^2z = a^m \cdot b^{2r} \cdot b^{m-k} b^m = a^m b^{m+r} \text{ not in } L$$

3.  $y$  consists of one or more  $a$ 's followed by one or more  $b$ 's.

$$y=ab, x=a^{m-1}, z=b^{m-1}$$

$$xyz = a^{m-1} ab b^{m-1}$$

$$xy^2z = a^{m-1} abab b^{m-1} = a^m ba b^m \text{ not in } L$$

# Pumping lemma (strong)

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Let  $L$  be an infinite regular language over  $A$ . and suppose  $L$  can be accepted by DFA  $(M)$  with  $m$  states. Then for any string  $w \in L$  such that  $|w| \geq m$  there exist  $x, y, z \in A^*$  such that

1.  $w = xyz, y \neq \epsilon$
2.  $|xy| \leq m,$
3.  $|y| \geq 1,$
4.  $w_i = xy^i z \in L$  for all  $i \geq 0$



# Example 1

---

$$L = \{a^n b^n : n \geq 0\}$$

Let  $L$  be is regular and suppose  $L$  can be accepted by DFA ( $M$ ) with  $m$  states. Then for any string  $w \in L$  such that  $|w| \geq m$  there exist  $x, y, z \in A^*$  such that

$$w = a^m b^m \quad |w| = 2m > m$$

By pumping lemma  $w = xyz = a^m b^m$

# Example 1

---

$$w = a^m b^m \quad |w| = 2m > m$$

By pumping lemma  $w = xyz = a^m b^m$

$$y = a^r, \quad r > 0, \quad |y| > 0$$

$$|xy| = k \leq m$$

$$x = a^{k-r}, \quad y = a^r, \quad z = a^{m-k} b^m$$

$$xy^2z = a^{k-r} \cdot a^{2r} \cdot a^{m-k} b^m = a^{m+r} b^m$$

*not in L*, Then L is not regular language

## Example 2

---

$L = \{a^n : n \text{ is prime}\}$  is not regular language

Let  $L$  be is regular and suppose  $L$  can be accepted by DFA ( $M$ ) with  $m$  states. Then for any string  $w \in L$  such that  $|w| \geq m$  there exist  $x, y, z \in A^*$  such that

$$w = a^m, |w| = m \geq m$$

By pumping lemma  $w = xyz = a^m$

# Example 2

---

$$w = a^m, \quad |w| = m = m$$

By pumping lemma  $w = xyz = a^m$ ,  $m$  is prime

$$y = a^r, \quad r > 0, \quad |y| > 0$$

$$|xy| = k \leq m$$

$$x = a^{k-r}, \quad y = a^r, \quad z = a^{m-k}$$

$$w_2 = xy^2z = a^{k-r} \cdot a^{2r} \cdot a^{m-k} = a^{m-k}$$

$$w_{m+1} = xy^{m+1}z = a^{k-r} a^{(m+1)r} \cdot a^{m-k}$$

## Example 2

---

$$w_{m+1} = xy^{m+1}z = a^{k-r} a^{(m+1)r} \cdot a^{m-k}$$
$$= a^{mr+m} = a^{m(r+1)}$$

*$r > 0$  then  $(r+1) > 1$  and  $m$  is prime then  $m > 1$*

*not in L, because  $m(r+1)$  not prime*

*Then L is not regular language*

