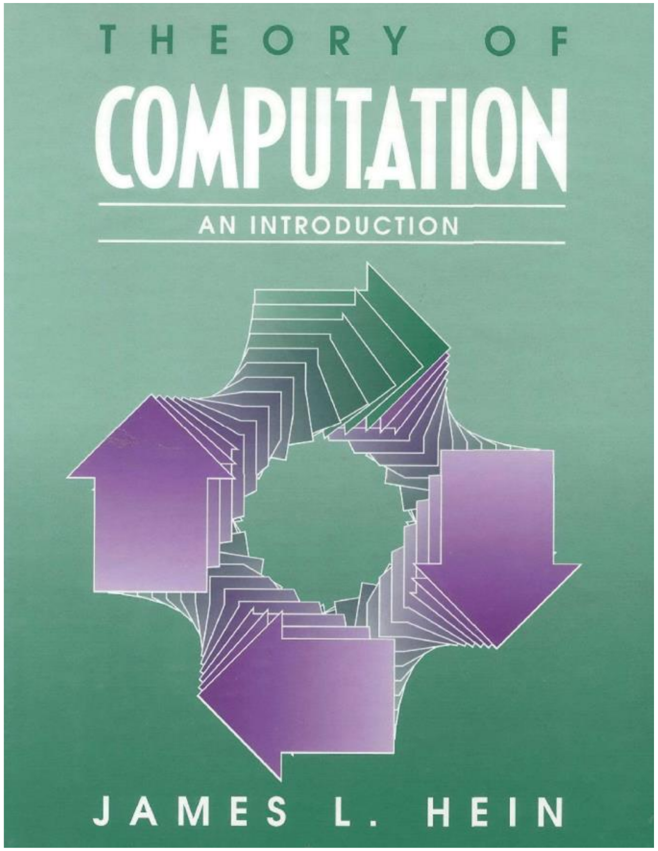
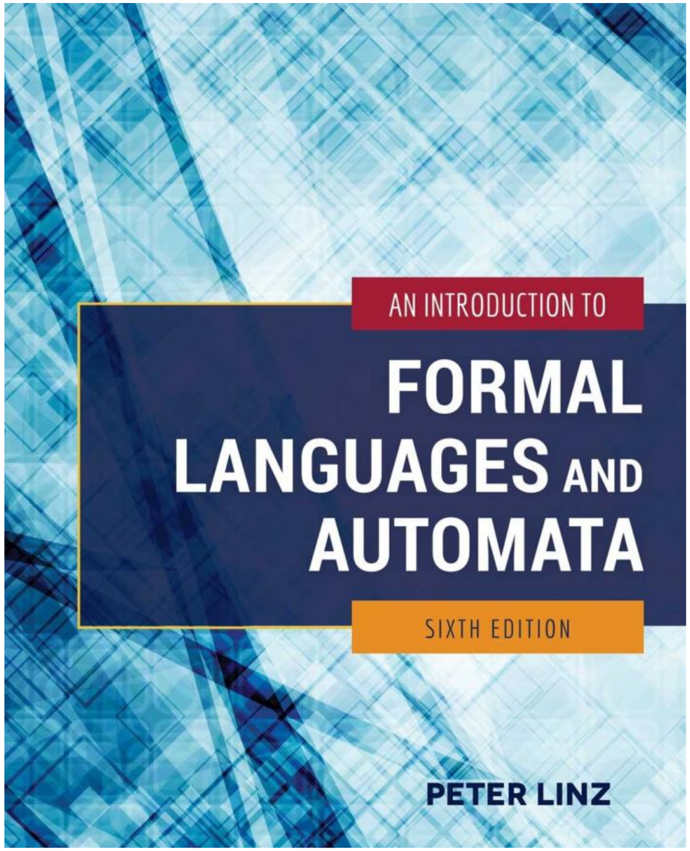
Automata and Formal Languages

Lecture 09

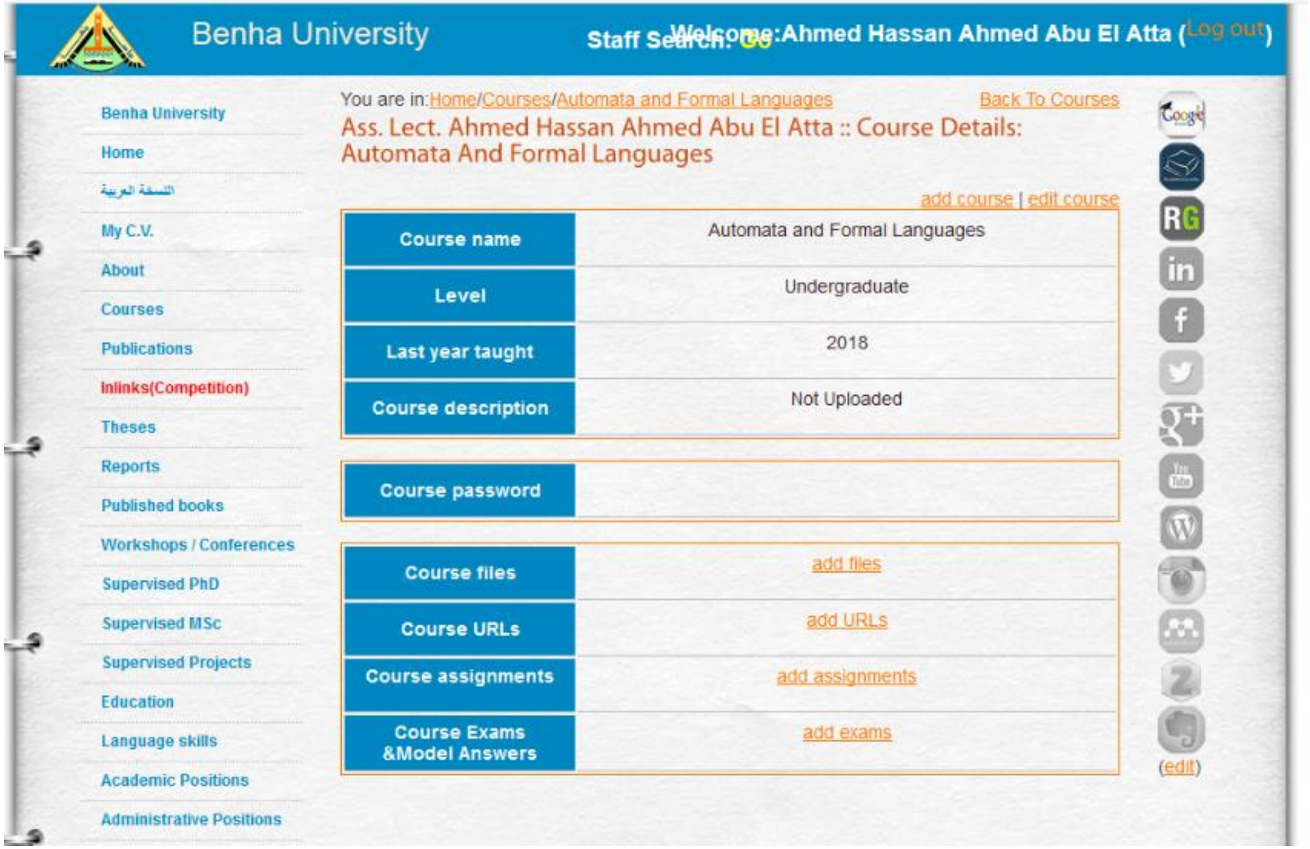
Books


THEORY OF
COMPUTATION
AN INTRODUCTION
JAMES L. HEIN


AN INTRODUCTION TO
FORMAL LANGUAGES AND AUTOMATA
SIXTH EDITION
PETER LINZ

PowerPoint

http://www.bu.edu.eg/staff/ahmedaboalatah14-courses/14767



Pumping lemma

PUMPING LEMMA

Agenda

- Pumping lemma1
- Example 1
- Pumping lemma2
- Example 2
- Example 3

Non-regular languages

(PUMPING LEMMA)

Non-regular languages

L={$a^n b^n$: n ≥ 0}

L={vv^R: v∈{a, b}*}

Regular languages

a^*b

$b^*c + a$

$b + c(a + b)^*$

etc...

How can we prove that a language L is not regular?

Prove that there is no DFA or NFA or RE that accepts L

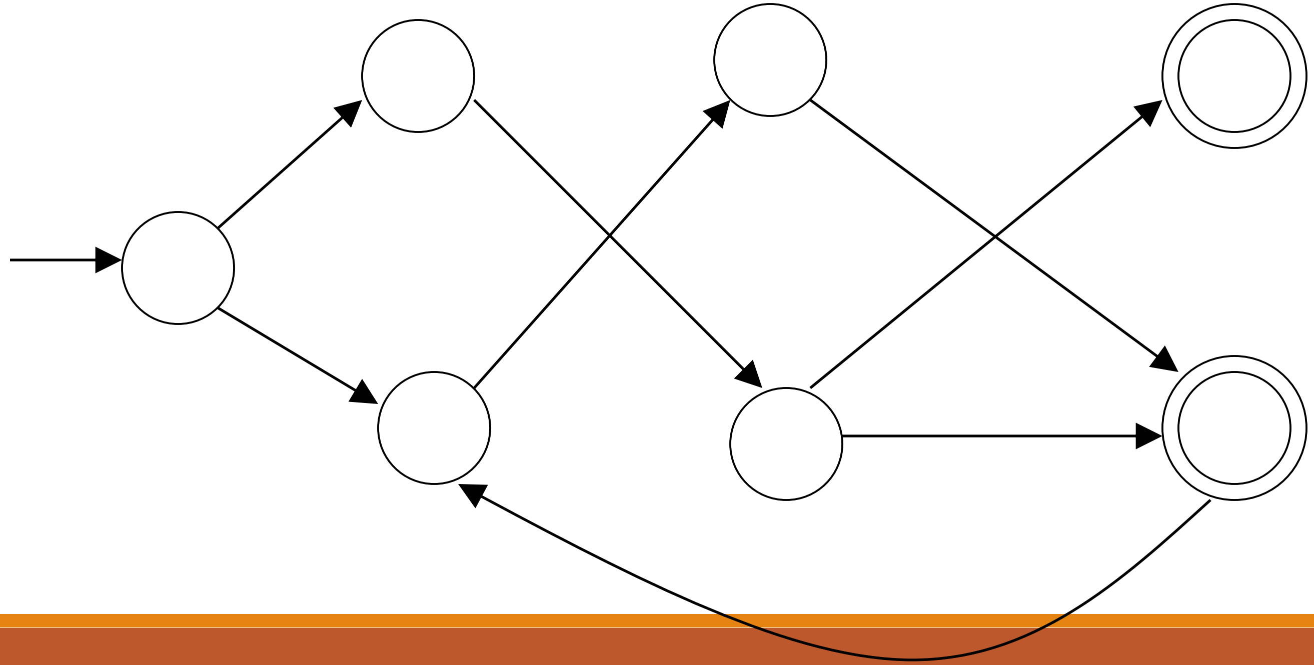
Difficulty: this is not easy to prove (since there is an infinite number of them)

Solution: use the Pumping Lemma !!!

Take an infinite regular language L

(contains an infinite number of strings)

There exists a DFA that accepts L



m
states

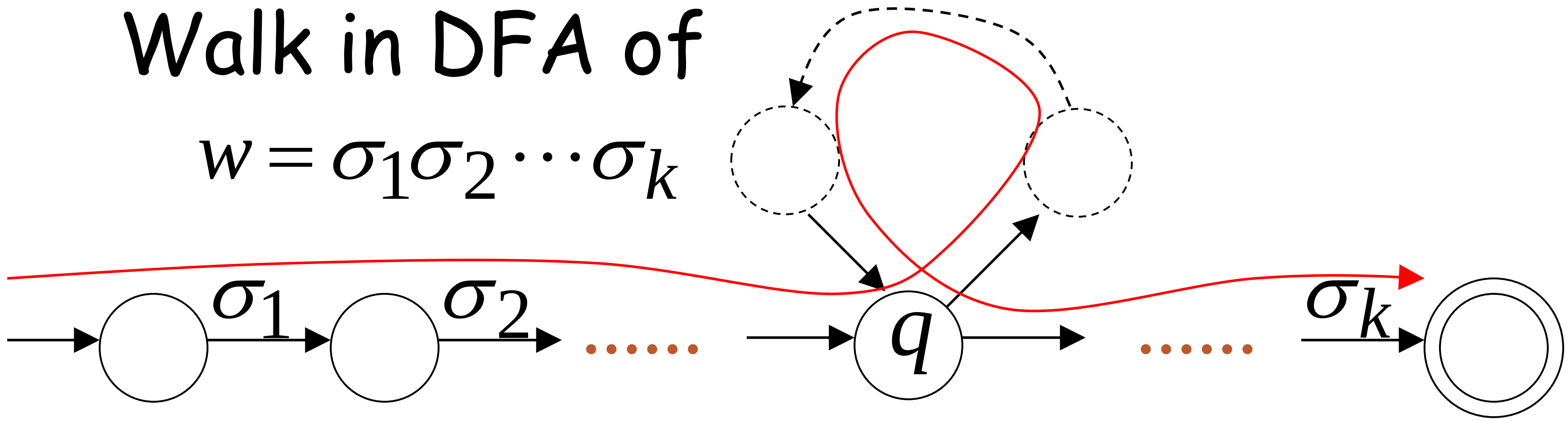
Take string $w \in L$ with $|w| \geq m$

(number of states of DFA)

then, at least one state is repeated in the walk of w

Walk in DFA of

$w = \sigma_1 \sigma_2 \cdots \sigma_k$

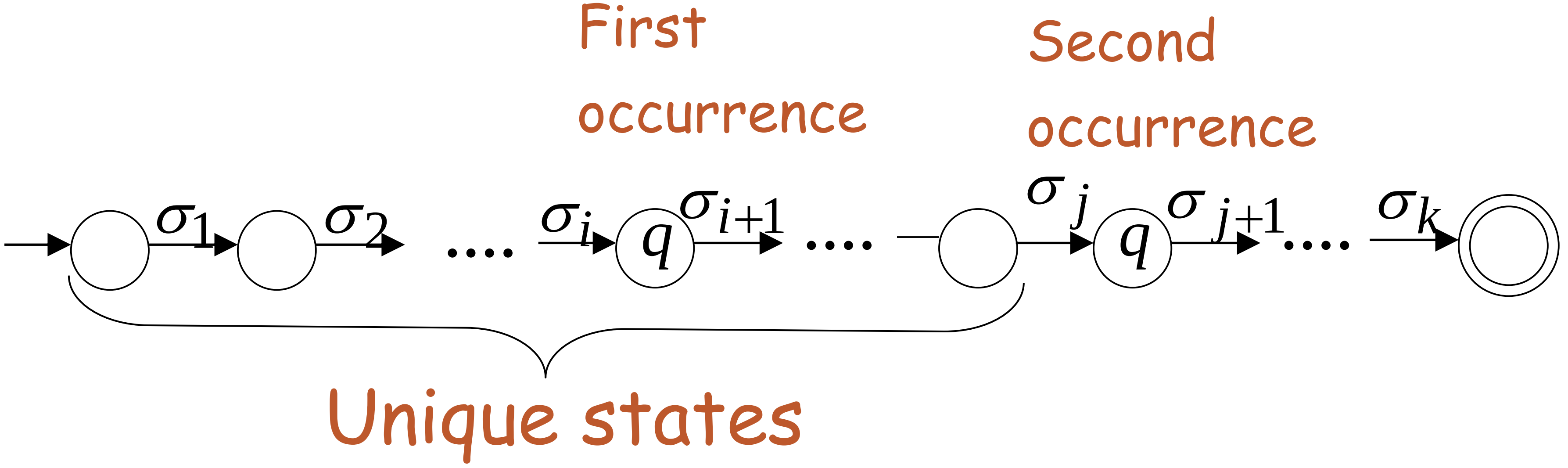


Repeated state in DFA

There could be many states repeated

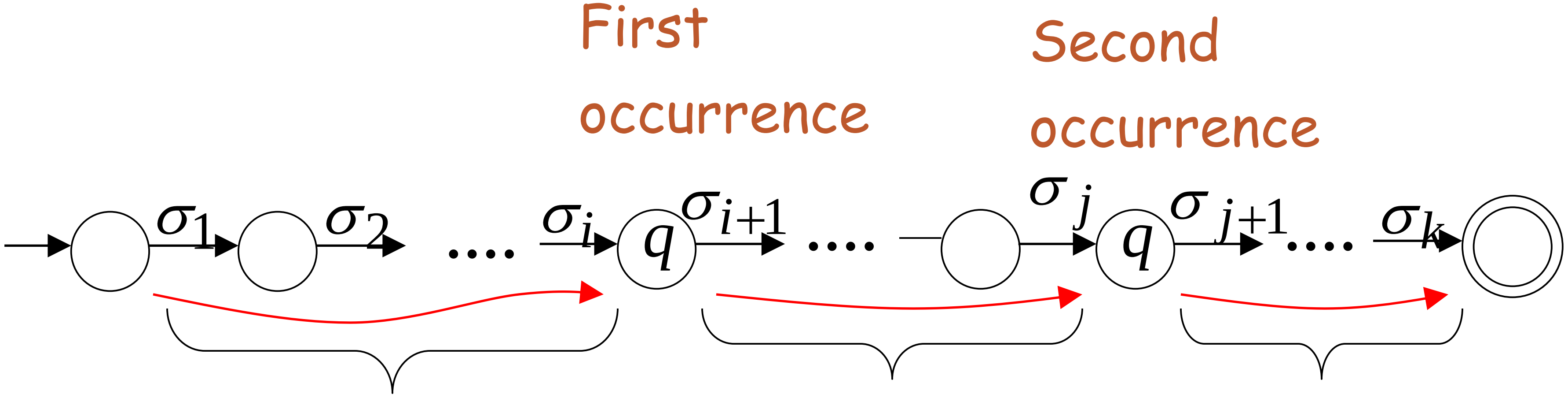
Take q to be the first state repeated

One dimensional projection of walk w :



We can write $w = xyz$

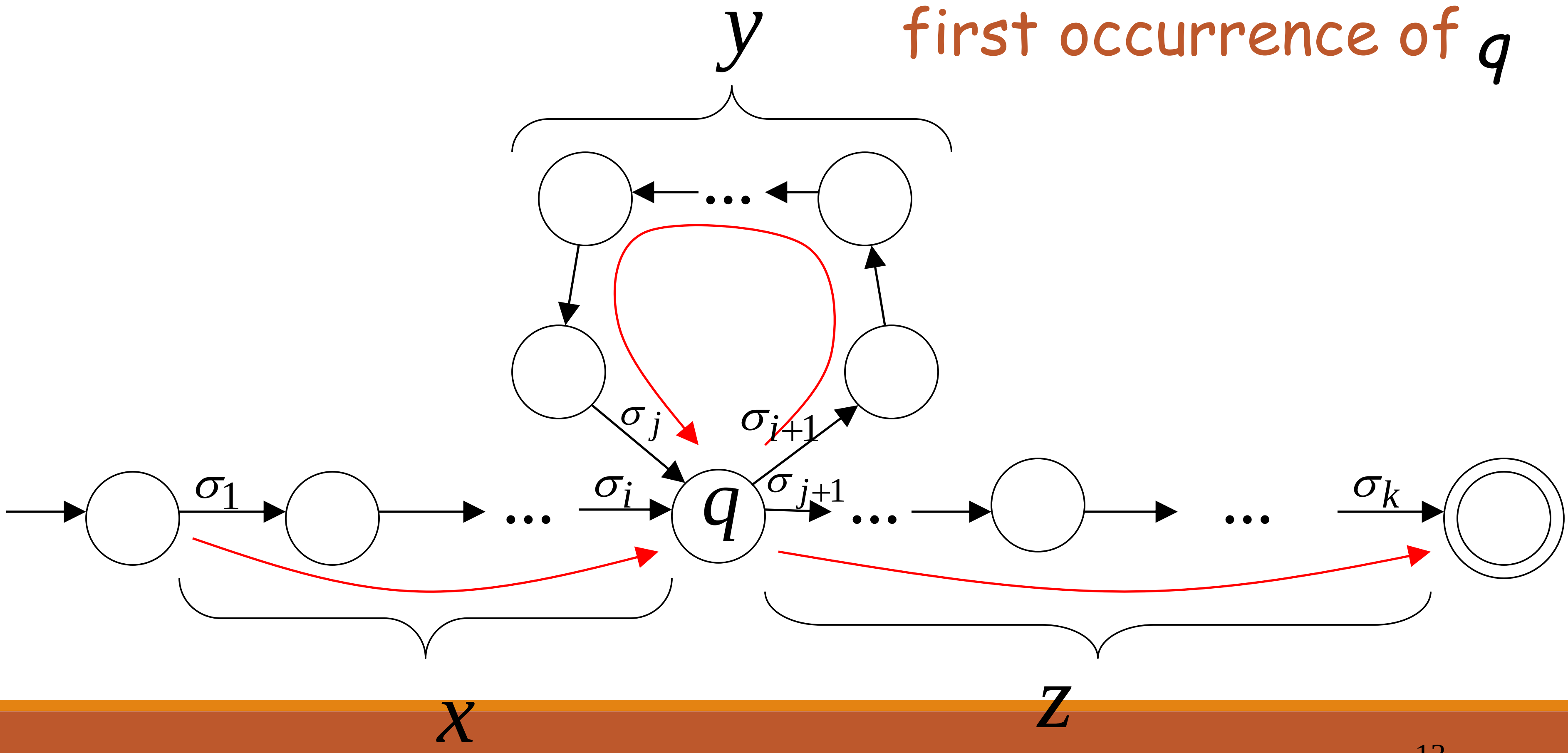
One dimensional projection of walk w :



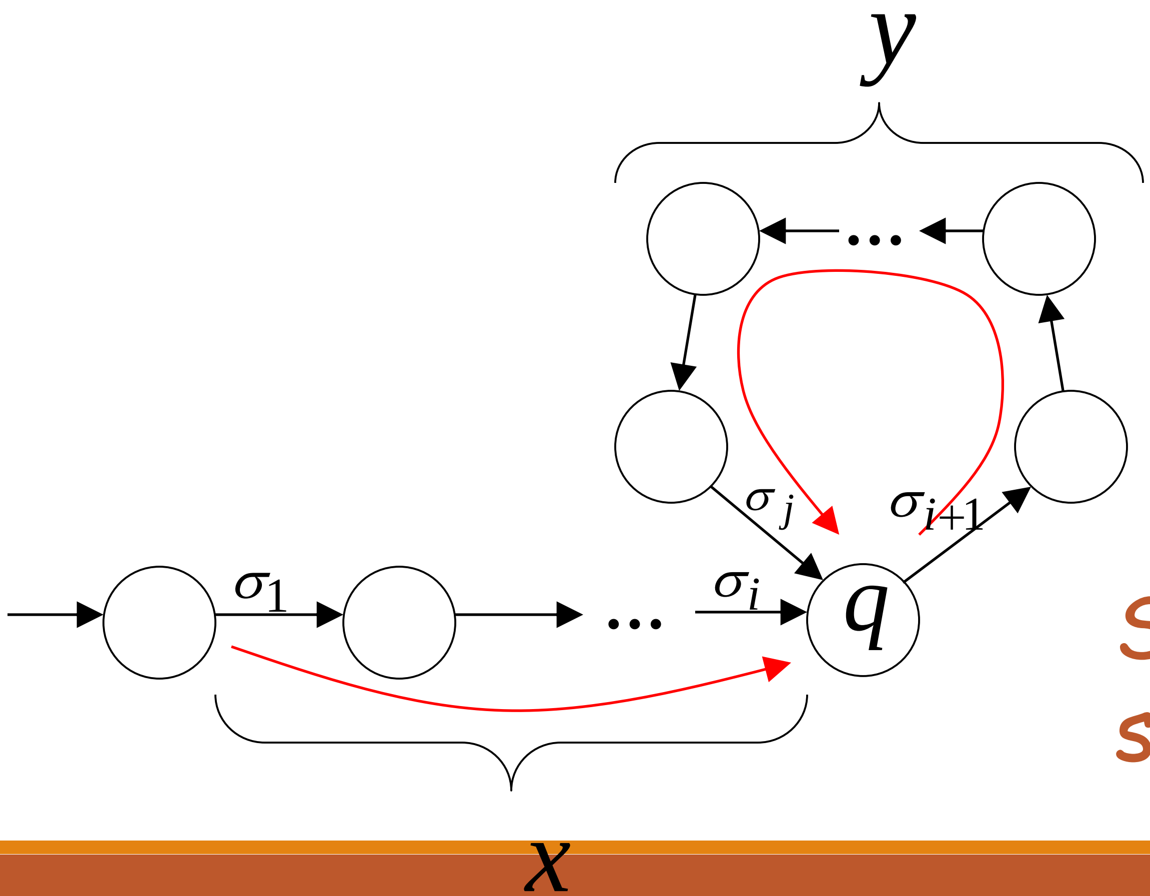
$x = \sigma_1 \cdots \sigma_i$ $\quad$ $y = \sigma_{i+1} \cdots \sigma_j$ $\quad$ $z = \sigma_{j+1} \cdots \sigma_k$

In DFA: $w = x\ y\ z$

contains only first occurrence of q



Observation: length $|x\,y| \leq m$ number of states of DFA

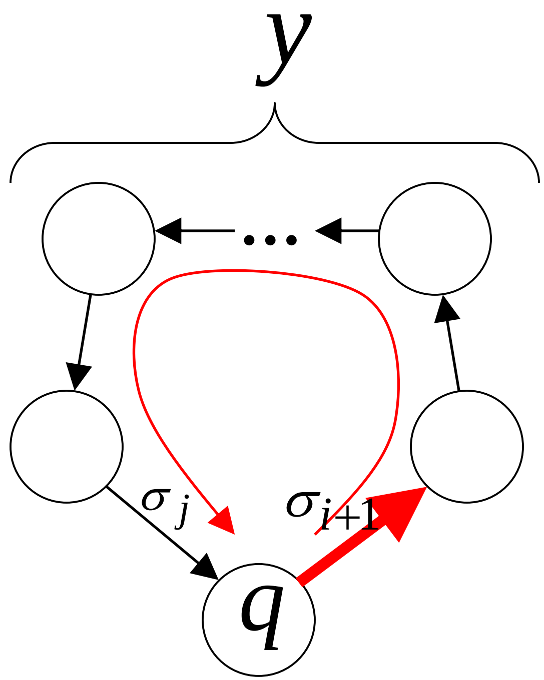


Unique States

Since, in xy no state is repeated (except q)

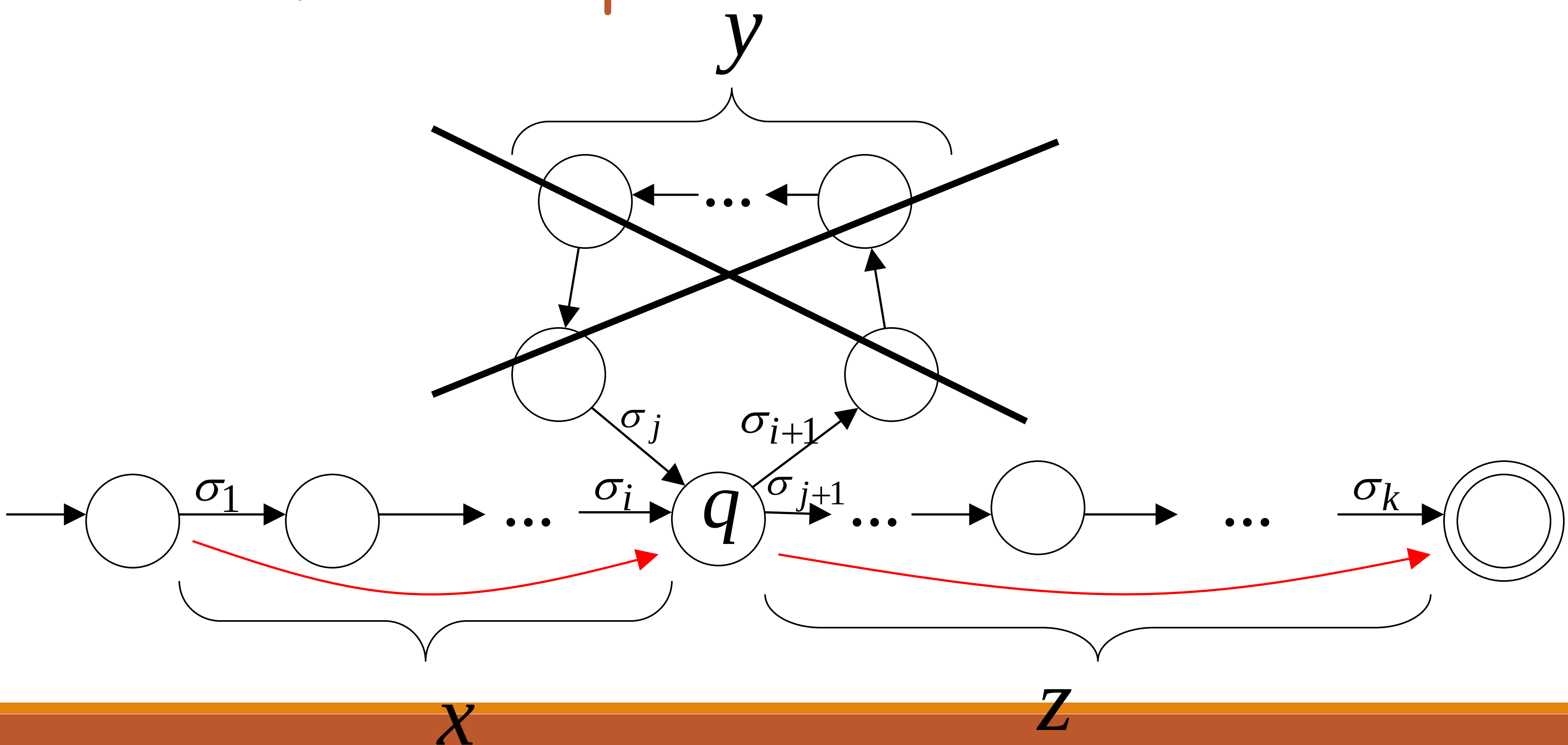
Observation: length $|y| \geq 1$

Since there is at least one transition in loop



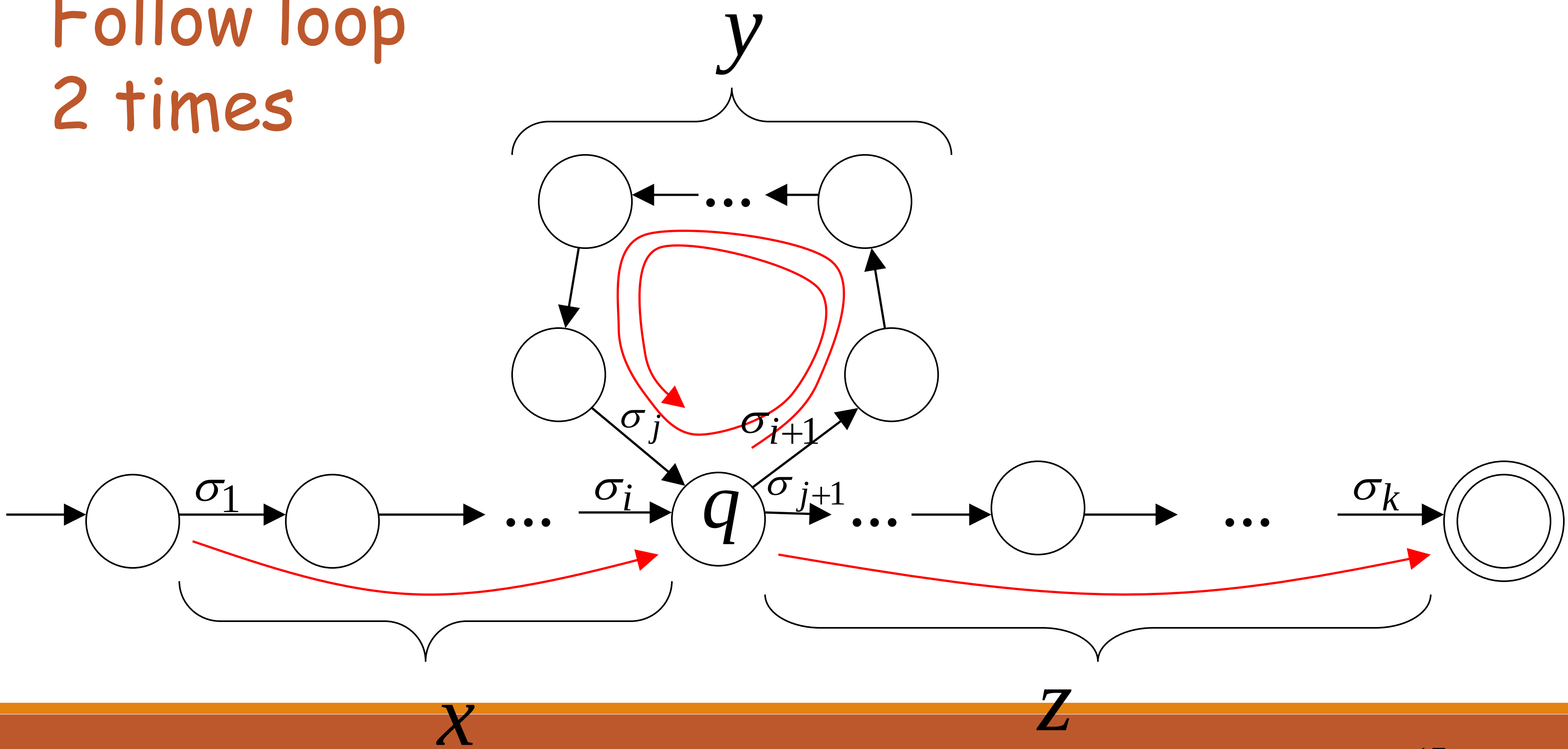
Additional string: The string $x\,z$ is accepted

Do not follow loop



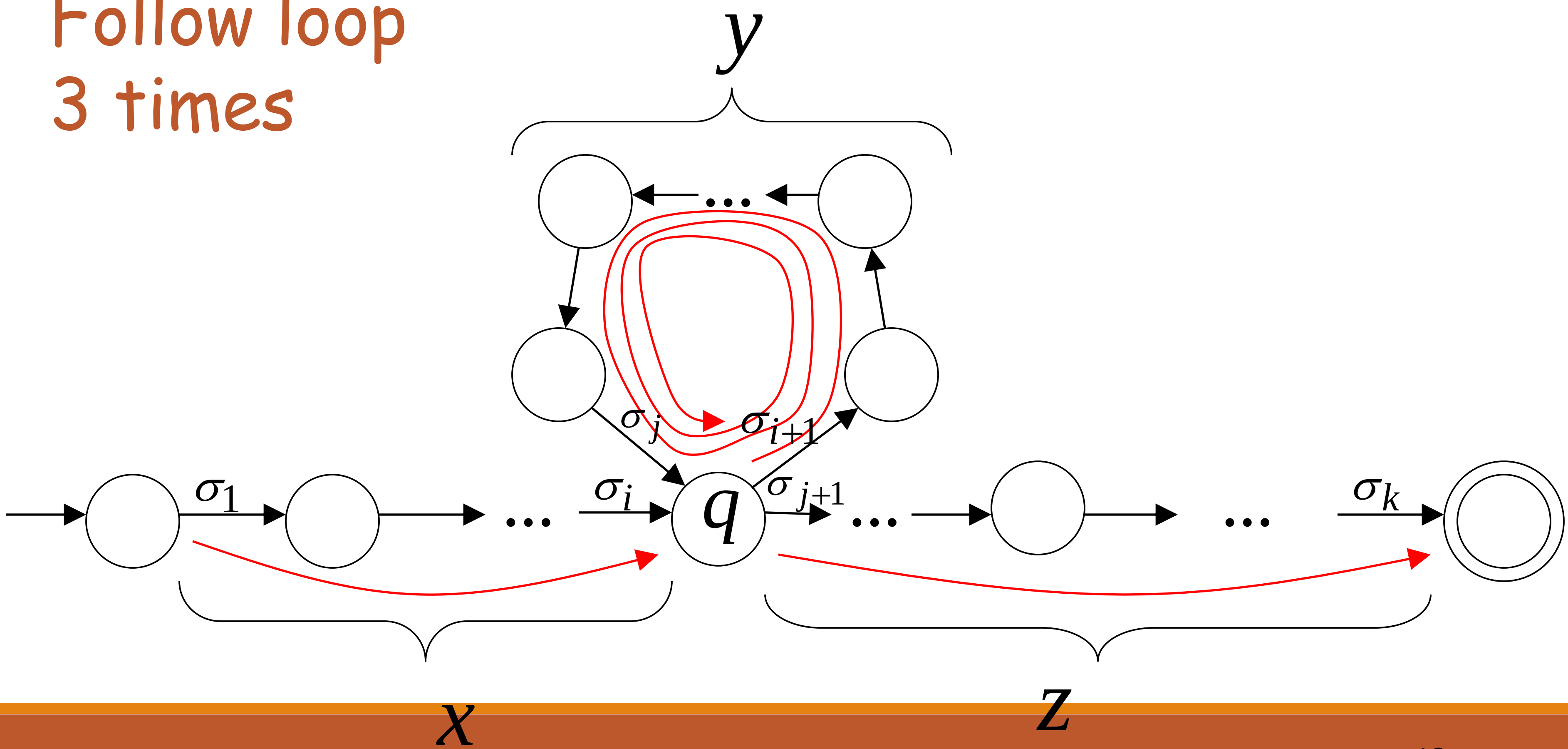
Additional string: The string $x\ y\ y\ z$ is accepted

Follow loop 2 times



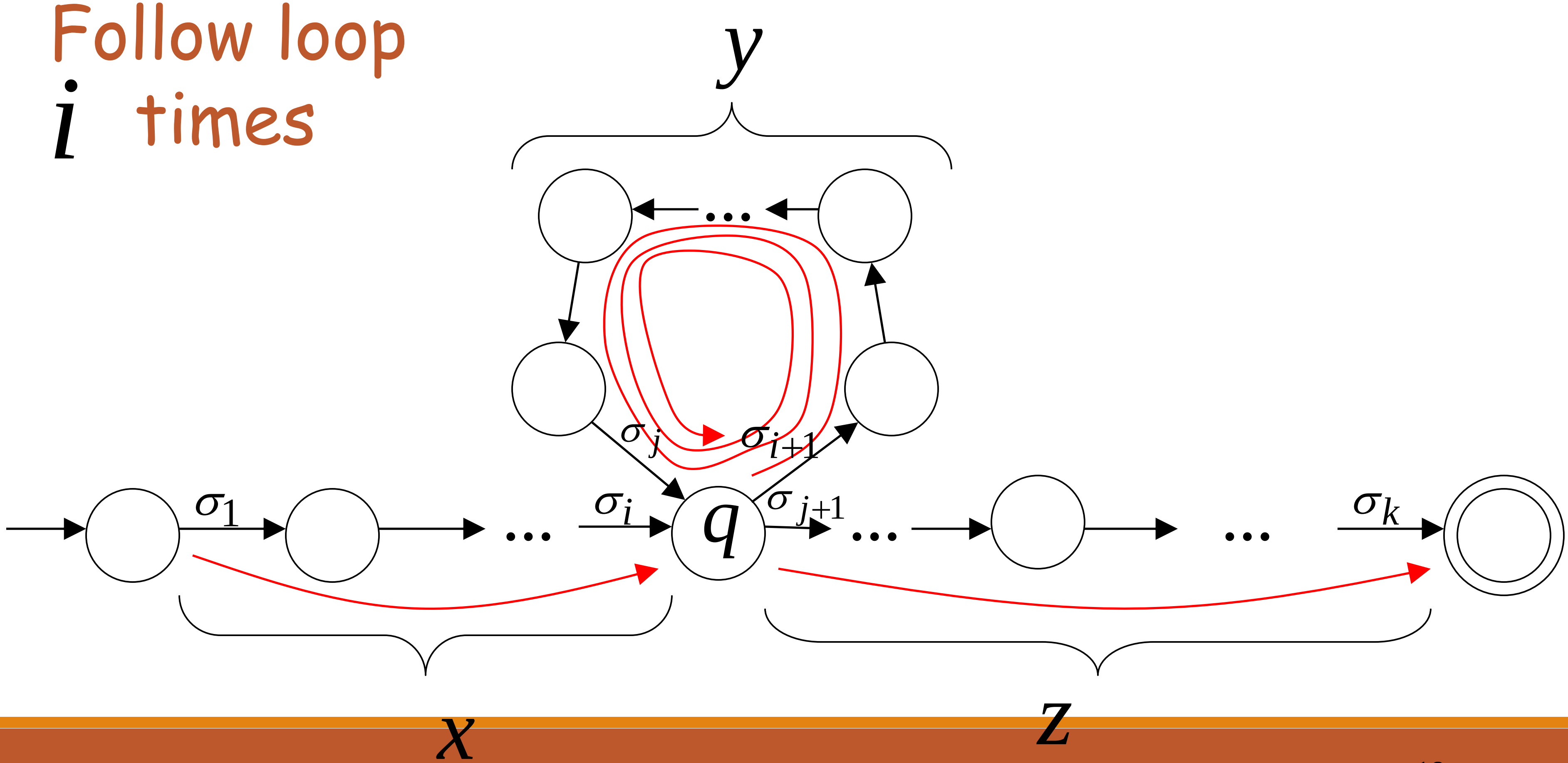
Additional string: The string $x\,y\,y\,y\,z$ is accepted

Follow loop 3 times



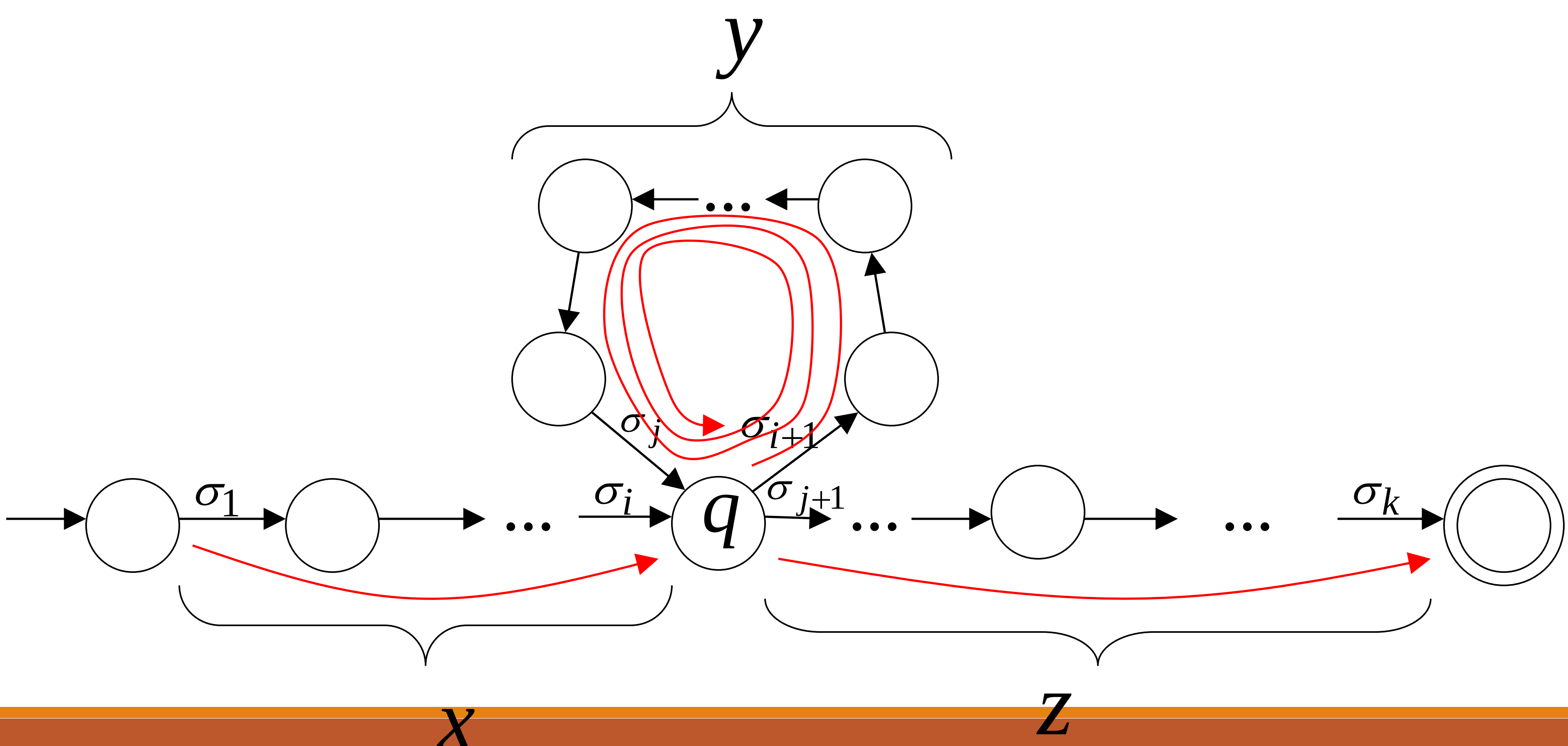
In General: The string $x\,y^i\,z$ is accepted $i = 0, 1, 2, \ldots$

Follow loop i times



Therefore: $x\,y^i\,z \in L$ $\quad i = 0, 1, 2, \ldots$

Language accepted by the DFA



Pumping lemma(weak)

Let L be an infinite regular language over A. then there exist strings

x, y, z ∈ A*, where y ≠ Ɛ,

such that

xy^iz ∈ L for all i ≥ 0. such that

Example 1

$L=\{a^n b^n: n \geq 0\}$

Let L be is regular and suppose L can be accepted by DFA (M) . Then there exist x , y, z ∈ A^* , let i=1

By pumping lemma $w = xyz = a^m b^m$

1. y consists of a's. |xy|=k, y=a^r. xy=a^k

$xy^2z = a^{m-r} . a^{2r} . a^{m-k} b^m = a^{m+r} b^m$ is not in L

Example 1

2. y consists of b's.

$xy^2z = a^m .b^{2r}. b^{m-k} b^m = a^m b^{m+r}$ not in L

3. y consists of one or more a's followed by one or more b's.

y=ab, x=a^{m-1}, z= b^{m-1}

$xyz = a^{m-1}$ ab b^{m-1}

$xy^2z = a^{m-1}$ abab b^{m-1}= a^m ba b^m not in L

Pumping lemma (strong)

Let L be an infinite regular language over A. and suppose L can be accepted by DFA (M) with m states. Then for any string w ∈ L such that |w| ≥ m there exist x , y, z ∈ A^* such that

1. w = xyz , y≠ Ɛ
2. |xy| ≤ m,
3. |y| ≥ 1,
4. w_i = xy^iz ∈ L for all i ≥0

Example 1

L={$a^n b^n$: n ≥ 0}

Let L be is regular and suppose L can be accepted by DFA (M) with m states. Then for any string w ∈ L such that |w| ≥ m there exist x , y, z ∈ A^*such that

w=$a^m b^m$ $|\mathrm{w}| = 2\mathrm{m} > \mathrm{m}$

By pumping lemma $w = xyz = a^m b^m$

Example 1

w=$a^m b^m$ $|\mathrm{w}| = 2\mathrm{m} > \mathrm{m}$

By pumping lemma $w = xyz = a^m b^m$

y=a^r , r >0, |y|>0

$|\mathrm{xy}| = \mathrm{k} \leq m$

x= a^{k-r} , y= a^r, z= $a^{m-k} b^m$

$\mathrm{x}y^2\mathrm{z} = a^{k-r} . a^{2r} . a^{m-k} b^m = a^{m+r} b^m$

$not\ in$ L, Then L is not regular language

Example 2

L={a^n: n is prime} is not regular language

Let L be is regular and suppose L can be accepted by DFA (M) with m states. Then for any string w ∈ L such that |w| ≥ m there exist x , y, z ∈ A^*such that

w=a^m , $|\mathrm{w}| = \mathrm{m}$ ≥ m

By pumping lemma w=$xyz = a^m$

Example 2

w=a^m , $|w| = m$=m

By pumping lemma $w = xyz = a^m$, m is prime

y=a^r , r >0, |y|>0

$|xy| = k \leq m$

x= a^{k-r} , y= a^r, z= a^{m-k}

w_2=xy^2z $= a^{k-r} .a^{2r}. a^{m-k}$=$a^{m-k}$

w_{m+1}=xy^{m+1}z $= a^{k-r} a^{(m+1)r}. a^{m-k}$

Example 2

w_{m+1}=xy^{m+1}z $= a^{k-r}a^{(m+1)r}.a^{m-k}$

=a^{mr+m}=$a^{m(r+1)}$

r>0 then (r+1)>1 and m is prime then m>1

not in L, because m(r+1) not prime

Then L is not regular language